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Ray Division Problems and Central Angle Cosine Problems: A Preliminary Discussion

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Abstract

According to the situation that the ray divides the space equally, the cosine of the angle between the two rays in this case is calculated. On this basis, the Ray Division Problem (RDP), Ray Division Problem without Number Constricts (RDPNC), Central Angle Cosine Sequence Problem (CACSP), and Central Angle Cosine Problem (CACP) are formed. Finally, through calculation, it is concluded that the CACP is correct.

Keywords: Ray Division Problem; Central Angle Cosine Problem; Regular Tetrahedron.

1. Introduction

Space bisected by rays is interesting, just like a laser beam. But its mathematical principles and numerical relationships are rarely mentioned. And the angle between the center of a shape and two vertices does not rise to the angle of the ray bisecting space. This seems to affect spatial geometry and physical science. This may be a preliminary research form of a theory.

2. Ray Bisection Problem

Ray Division Problem (RDP). An m-dimensional space is averagely divided into n=m+1 parts by p=m+1 rays. The cosine value of the angle between any two such rays starting from the center of the space is -1/m.

There is a constraint here, that is, p=m+1. If p>=m+1, it can evolve into another problem.

Let's briefly discuss some of the following situations:

Case of m = 1:

An m (m=1) dimensional space (straight line) is averagely divided into n=m+1=2 rays and divided into p=m+1=2 parts. The cosine of the angle formed by the two rays is -1/m.

Case of m = 2:

An m (m=2) dimensional space (plane) is averagely divided into n=m+1=3 rays and divided into p=m+1=3 parts. The cosine of the angle formed by any two rays starting from the center point of the equilateral triangle is -1/m.

Case of m = 3:

An m (m=3) dimensional space (solid (sphere, regular tetrahedron)) is averagely bisected into n=m+1=4 parts by p=m+1=4 rays. The cosine of the angle formed by any two rays starting from the center point of the regular tetrahedron is -1/m.

When m>3:

It is not possible to discuss and calculate multidimensional space at this time.

Hypothesis 1. Is there a phenomenon of high-dimensional collapse? In other words, multidimensional space is unobservable? The only observable dimension is 3-dimensional space. Once observed, the high-dimensional space will collapse into 3-dimensional space. Is this similar to quantum mechanics?

Hypothesis 2 (Opposite of RDP). High-dimensional space cannot be divided equally?

But the above content is not the focus of this article. This article will discuss some of its evolution issues.

3. Central Angle Cosine Problem

Ray Division Problem without Number Constricts (RDPNC). An m-dimensional space is divided into n>=m+1 parts by p>=m+1 rays. The cosine value of the angle between any two such rays starting from the center of the space is -1/q (q>=m), q=m, m+1, ….

Here, we relax the constraint and the number of rays can be greater than m+1. This means that even in 3-dimensional space, there can be more than 4 rays that can evenly divide the space. In this case, we can discuss another problem, i.e.,

Central Angle Cosine Sequence Problem (CACSP). In m (m<=3) dimensional space, what shapes have a cosine of -1/q (q>=m) between the center of the body and any two vertices, when q=m, m+1, ….?

Central Angle Cosine Problem (CACP). In m (m<=3) dimensional space, what shapes have a cosine of -1/q (q=m) between the center of the body and any two vertices, when q=m?

In this article, we only study and discuss the Central Angle Cosine Problem (CACP). In our next articles, we will discuss the Central Angle Cosine Sequence Problem (CACSP).

4. Computation and Exploration of Central Angle Cosine Problem (CACP)

4.1. Case of m = 1, q = m:

In 1-dimensional space, that is, on a straight line, we send out two rays at the center point (assuming any point as the center). The angle θ between them and the center point is 180°, $\cos\theta = -\frac{1}{1} = -1$, as seen in Fig.1(a).

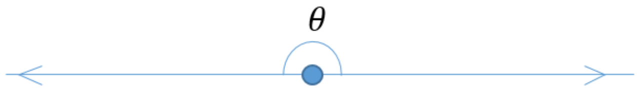
4.2. Case of m = 2, q = m:

In 2-dimensional space, i.e. on a plane, we emit two rays at the center point (arbitrarily assume that a point is the center of an equilateral triangle). The angle θ between them and the center point is 120°, $\cos\theta = -\frac{1}{2}$, as seen in Fig.1(b).

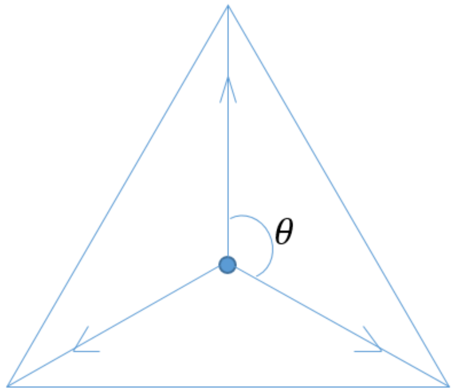
4.3. Case of m = 3, q = m:

After preliminary guessing. The cosine of the angle between the center of a regular tetrahedron [1] and any two vertices is -1/3 [2], as seen in Fig.1(c).

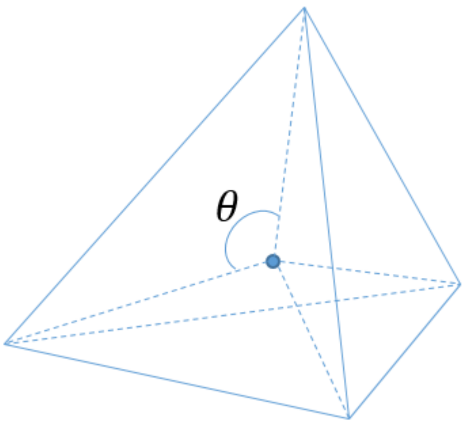
The regular tetrahedron is the only Platonic solid with equal distances between all vertices. It is also the only way to make the distances between every two vertices of four vertices equal in three-dimensional space. The regular tetrahedron is a three-dimensional regular simplex (3-simplex). In a higher-dimensional hyperspace, any four vertices must be in the same three-dimensional space. If these four vertices do not have four coplanar points, three collinear points, or two coincident points, they can definitely form a tetrahedron. [1]



(a) Case of m = 1, q = m (Straight Line)



(b) Case of m = 2, q = m (Equilateral Triangle)



(c) Case of m = 3, q = m (Regular Tetrahedron)

Fig.1 Geometric interpretation of CACP

Let's start calculation verification:

Let the vertices of the regular tetrahedron be A, B, C, D, and find the cosine of the angle between the center O and the vertices. We choose a suitable set of coordinate points to make the calculation simple. Common regular tetrahedron coordinate settings are as follows:

$$A=(1,1,1), B=(1,-1,-1), C=(-1,1,-1), D=(-1,-1,1)$$

$$O=\left(\frac{1+1+(-1)+(-1)}{4},\frac{1+(-1)+1+(-1)}{4},\frac{1+(-1)+(-1)+1}{4}\right)$$

The formula for the cosine of the angle between vectors is:

$$\cos\theta=\frac{\overrightarrow{OA}\cdot\overrightarrow{OB}}{\left|\overrightarrow{OA}\right|\left|\overrightarrow{OB}\right|}$$

So the cosine value is:

$$\cos\theta = \frac{-1}{\sqrt{3}\times\sqrt{3}} = -\frac{1}{3}$$

5. Conclusion

We can conclude that the Central Angle Cosine Problem (CACP) is correct. CACP is easy to prove. But the Central Angle Cosine Sequence Problem (CACSP) is not easy to prove. I will give the calculation of several sequences in the following paper. In addition, several of my hypotheses should also be inspiring, but not necessarily rigorous. In future research, we will discuss whether the space can be divided equally.

References

[1] Regular tetrahedron, https://en.wikipedia.org/wiki/Tetrahedron#Regular_tetrahedron

[2] Brittin, W. E. (1945). Valence angle of the tetrahedral carbon atom. Journal of Chemical Education. 22 (3): 145. doi:10.1021/ed022p145