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Research on Beamforming Technology of Sparse Distributed Antenna Array Based on Compressed Sensing

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Abstract

In response to complex wireless communication network environments, in order to improve algorithm stability, accelerate convergence speed, and reduce hardware costs in beamforming technology, this paper studies a beamforming technology based on compressed sensing, which uses compressed sampling and reconstruction to recover signals and then form beams. Firstly, an improved Orthogonal Matching Pursuit (OMP) algorithm and a self-designed SPGL1 algorithm group were designed. Then, the missing sampling part was simulated and reconstructed using the DCT measurement matrix, and the recovery function was called to recover the waveform and calculate the coefficients after sparse transformation. Finally, a sparse multi element sparse array beam was constructed, Then, it is compared with a uniform array of multiple elements and combined with the convex optimization CVX tool package to use the L1 norm minimum algorithm for beam reconstruction. The experimental results show that beam forming simulation based on the improved traditional OMP algorithm achieves better beam recovery.

Keywords: Adaptive Beamforming; Sample Matrix Inversion; Compressive Sensing; Sparse Distributed Antenna Array

1. Introduction

The beamforming algorithm for sparse distributed antenna arrays based on compressed sensing proposed in this article can greatly reduce the number of array elements without reducing the antenna diameter and ensuring beam efficiency[1-2]. It is a new sparse array element shaping method that significantly reduces the number of front-end RF components. It utilizes the sparsity of the echo beam signal in the spatial domain, and then selects a randomly arranged array unit with a diameter to receive it[4-6]. Through a compression sensing strategy, the channel values are restored when the array is full rank, and an adaptive digital beam array is formed using the restored parameters when the array is full rank. The simulation results indicate that the algorithm theory has broad development potential.

2. Basic Principles of Compressed Sensing

The CS theory proposed by scholars E. Candès is an emerging dimensionality reduction sampling technique[7-9]. According to the sparsity of the signal in space, it is different from the traditional Nyquist sampling law, which first samples, stores, and compresses the signal at twice the frequency. Instead, it compresses the data during the data collection process, and this compression ensures low distortion recovery of the original data, thereby reducing the storage space and computational complexity of signal sampling. The use of compressed sensing reconstruction technology can reduce the number of antennas, reduce the sampling, storage, and processing costs of antenna systems, and effectively solve the problems of traditional beamforming technology[10].

Firstly, signal X (t) is represented as a sparse signal, which should be mapped to a sparse space through a matrix, represented as

$$X = \Psi S \tag{1}$$

Where, signal X is an N-dimensional vector, Ψ is a sparse transformation matrix of $N \times N$ dimensions. S is a $N \times 1$ dimensional vector after sparse representation X, where only K values are not zero, and the remaining values are all zero. K Is the sparsity of the signal, and $K < M$.

Then, perceptual measurement is performed on the sparse signal represented, Supposed Φ is a sampling matrix of $M \times N$ dimensions (M is the measurement value of the signal, and $M < N$), The projection vector of X on Φ is Y, and Y represents the measurement value obtained from M sampling bases less than N, and its dimension is $M \times 1$, then the observation vector Y is:

$$Y = \Phi X \tag{2}$$

Let P be the product of the sparse representation matrix and the sampling vector, and obtain the mapping matrix expression as follows:

$$P = \Phi \Psi \tag{3}$$

Therefore, the observation vector can also be expressed as:

$$Y = \Phi \Psi S = PS \tag{4}$$

The sampling reconstruction principle diagram of compressed sensing can be derived from formulas (1) - (3), as shown in Figure 1.

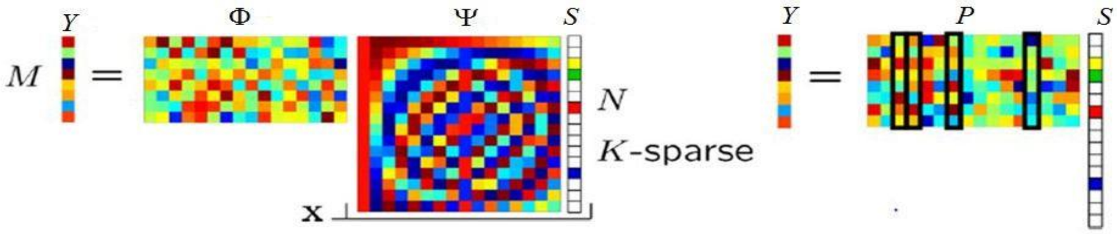

Fig. 1 Compressed Perception Sampling Reconstructed Graph

The principle of compressed sensing is used to reconstruct signals. After setting the input signal, the sampling matrix Φ, sparse basis Ψ, mapping matrix P, and observation matrix Y are known, and the CS algorithm needs to be used to calculate the coefficient S with sparsity K, which is then substituted into formula 1 to restore the original signal $X^{'}(t)$. Afterwards, we can use the reconstructed signal to form a beam.

3. Verification and Implementation of Simulation for 3 Compressed Sampling Reconstruction

3.1. Validation of Achievable Methods for Compression Reconstruction Technology

The simulation implementation of technology feasibility verification mainly includes three parts, namely generating signals, generating perception matrices and sparse representation matrices, and reconstructing beam signals.

Firstly, the signal generation mainly utilizes trigonometric functions to generate discrete signals in the frequency domain or DCT domain and draw signal beammaps; Then, the perception matrix and sparse representation matrix are generated, and the main process is to use the perception matrix to generate measurement values. Then, Fourier transform is performed on sparse orthogonal basis in the frequency domain, and the matrix is restored; Finally, the signal reconstruction process mainly involves using the convex optimization CVX toolkit to solve for the minimum L1 norm and using the orthogonal matching tracking OMP algorithm for reconstruction. The simulation results include both time and frequency domains, and the signal beam is generated as shown in Figure 2.

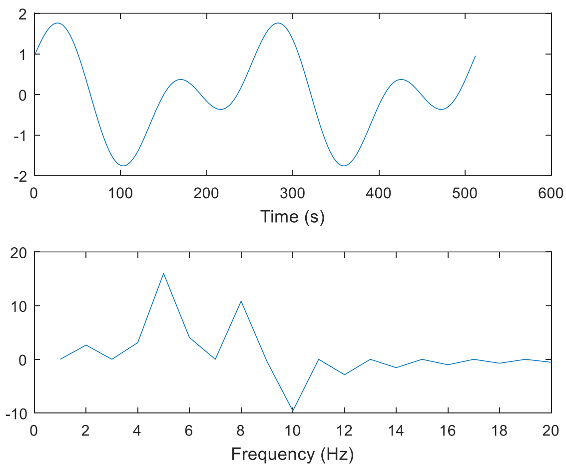

Fig.2 Generate signal beam

From Fig.2, it can be seen that the time-domain waveform is sparse, while the frequency-domain waveform is dense and tends to flatten relatively quickly. Combining the two methods can minimize errors. The initial and reconstructed waveforms for solving the minimum L1 norm are shown in Fig.3.

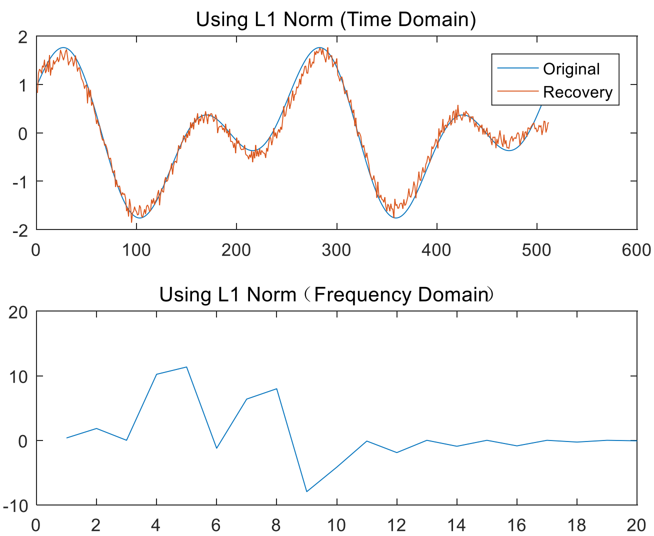

Fig. 3 Initial and reconstructed waveform for solving the minimum L1 norm

From Fig.3, it can be seen that the waveform reconstructed by combining the convex optimization CVX toolkit to solve the minimum L1 norm in the time domain presents a sawtooth shape, and fluctuates up and down on the basis of the original waveform. However, the waveform reconstructed using the orthogonal matching tracking OMP algorithm is relatively smooth, with relatively small errors for both. The orthogonal matching tracking OMP algorithm is used to reconstruct the waveform as shown in Fig. 4.

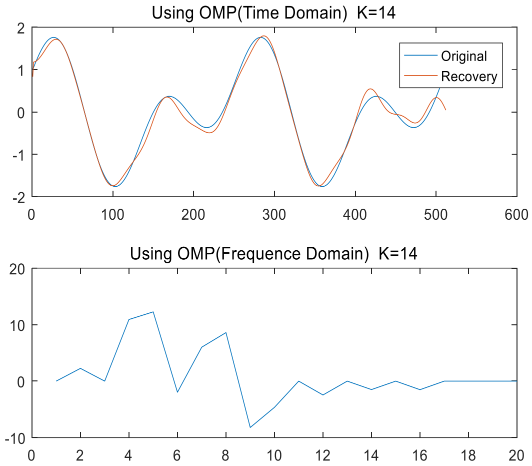

Fig. 4 OMP Algorithm for Reconstructing Waveforms

From Fig.4, it can be seen that the waveform reconstructed using the orthogonal matching tracking OMP algorithm in the frequency domain is basically consistent with the original signal waveform, while the waveform reconstructed by solving the minimum L1 norm using the convex optimization CVX toolkit shows small fluctuations at the far end of time.

3.2. Implementation Process of Compressed Sampling Reconstruction Simulation

On the basis of fully verifying the feasibility of orthogonal matching pursuit OMP algorithm reconstruction and combining convex optimization CVX toolkit to solve L1 norm minimum reconstruction algorithm, this design adopts the improved orthogonal matching pursuit OMP algorithm and the groundbreaking SPGL1 algorithm group, including but not limited to L1 norm minimum reconstruction algorithm, to achieve compressed sampling and reconstruction method for signal recovery.

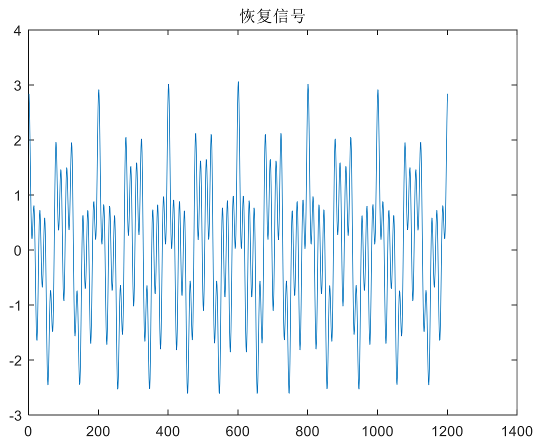

Fig.5 OMP Algorithm Recovery Signal Diagram

This design selects the superposition of sinusoidal signals with frequencies of 2, 5, and 13 Hz as the target signal, with a sampling frequency of 200 Hz and a random undersampling signal of 50 points. The undersampling DCT sequence is used to generate a DCT sparse transformation matrix, which in turn generates the corresponding compressed sensing measurement matrix. The improved OMP algorithm restores the signal, the original signal, and the random undersampling signal as shown in Fig.5.

From Fig.5, it can be intuitively seen that the OMP algorithm restores the signal waveform. The original signal is shown in Fig.6.

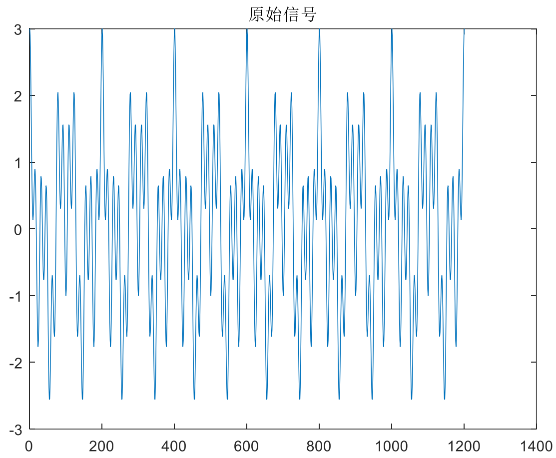

Fig.6 Original Signal Diagram of OMP Algorithm

From Fig.6, it can be intuitively seen that the original signal waveform of the OMP algorithm. The random undersampling signal is shown in Fig.7.

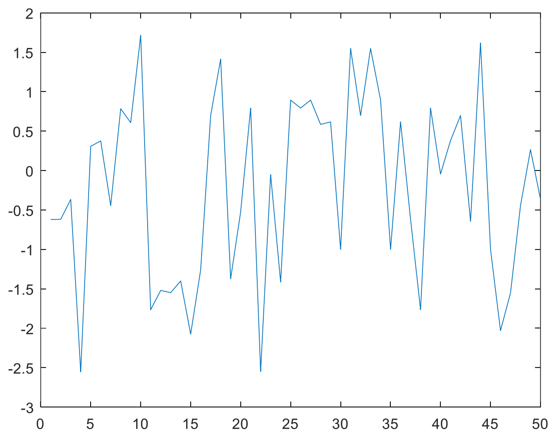

Fig.7 Random undersampling signal diagram of OMP algorithm

From Fig.7, it can be intuitively seen that the OMP algorithm randomly undersamples the signal waveform. From the above three images, it can be seen that the improved OMP algorithm achieves better signal beam recovery under undersampling conditions, with similar frequency and waveform compared to the original signal. The self designed SPGL1 algorithm for undersampling signals is shown in Fig.8.

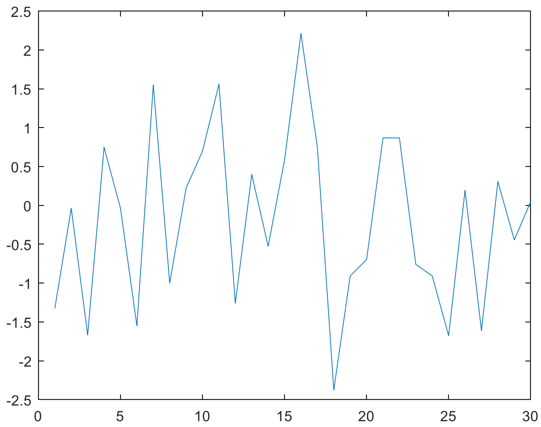

Fig.8 Undersampling Signal Diagram of SPGL1 Algorithm

From Fig.8, it can be intuitively seen that the SPGL1 algorithm randomly undersamples the signal waveform. The self-designed SPGL1 algorithm for signal recovery is shown in Fig.9.

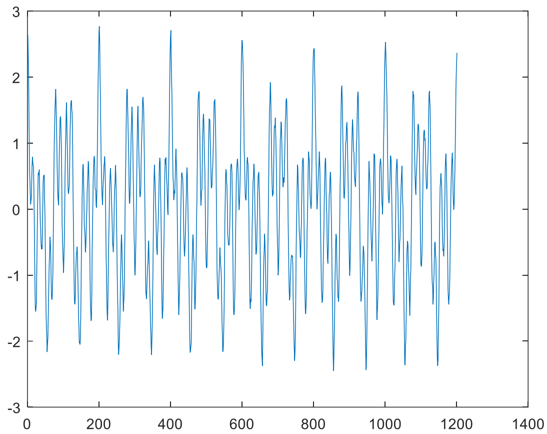

Fig.9 SPGL1 Algorithm Recovery Signal Diagram

From Fig.9, it can be intuitively seen that the SPGL1 algorithm restores the signal waveform. The original signal of the self-designed SPGL1 algorithm is shown in Fig.10.

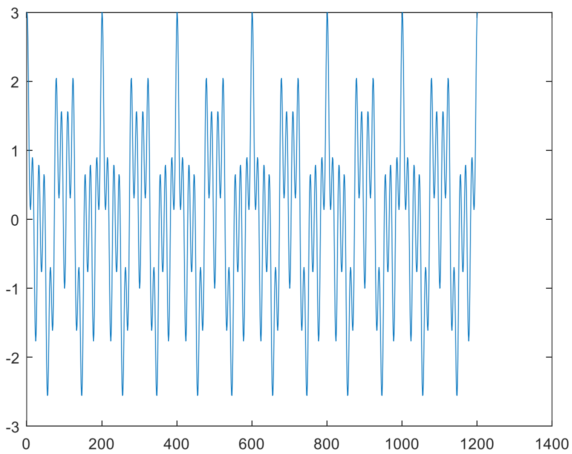

Fig. 10 Raw Signal Diagram of SPGL1 Algorithm

From Fig.10, it can be intuitively seen that the original signal waveform of the SPGL1 algorithm. From the above figure, it can be seen that the SPGL1 algorithm has also achieved good signal beam recovery under undersampling conditions. The difference is that the improved OMP algorithm increases the amplitude of the recovered waveform, while the self-designed SPGL1 algorithm decreases the amplitude.

3.3. Implementation Process of Compressed Sensing Beamforming

Select an array with 16 elements and set a sparse orthogonal matrix with a certain degree of sparsity. Then, use the Lagrange multiplier method to recover the sampling matrix. Finally, use the method of multiplying the uniform line element orthogonal matrix with the sparse sampling matrix to obtain the sparse sampling beamforming represented by the red line, Using the traditional sampling matrix inversion algorithm, the standard linear element beamforming represented by the blue line and the linear element beamforming with random errors represented by the green line can be obtained, as shown in Fig. 11.

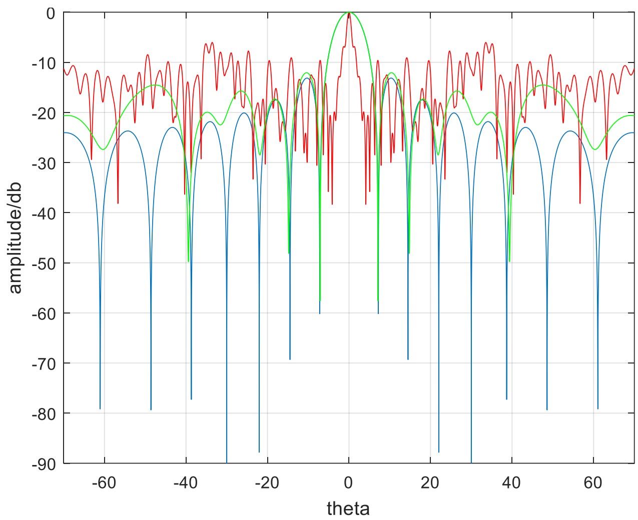

Fig.11 Direction Map of 16 Element Uniform Linear Array and Sparse Array

From Fig.11, it can be seen that under the same array element, the angle range occupied by the main lobe of the sparse array is small, and the number of sidelobes increases, but the array gain range decreases. In order to further clarify the sidelobe angle range of the sparsely distributed array with 16 elements, 82 elements of uniform linear array beamforming were selected for comparison. The simulation results are shown in Fig.12.

From Fig.12, it can be seen that the sidelobe direction range of the 16 element sparse array and the 82 element uniform linear array are similar, which can intuitively prove that the sparse array greatly reduces the beam radiation range. Similar to the time domain, based on the research on the frequency domain and sparse array mentioned above, the convex optimization CVX toolkit is used to solve the L1 norm minimum reconstruction algorithm for compressed sampling and reconstruction of the resulting beam, as shown in Fig.13.

From Fig.13, it can be seen that the CVX toolkit solves the norm method well, achieving the recovery of 16 element sparse arrays using the compressed sampling reconstruction method.

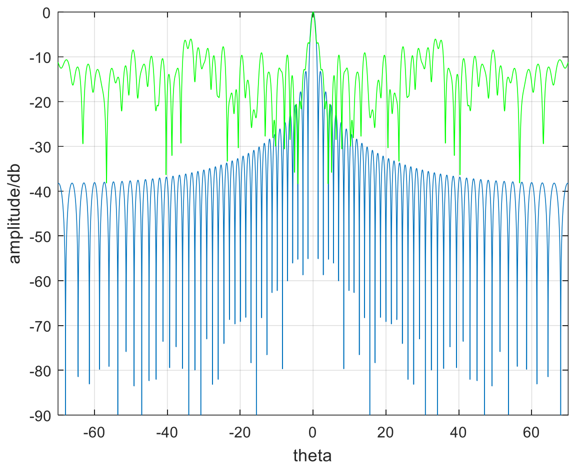

Fig.12 Comparison of 82 element uniform array and 16 element sparse array directions

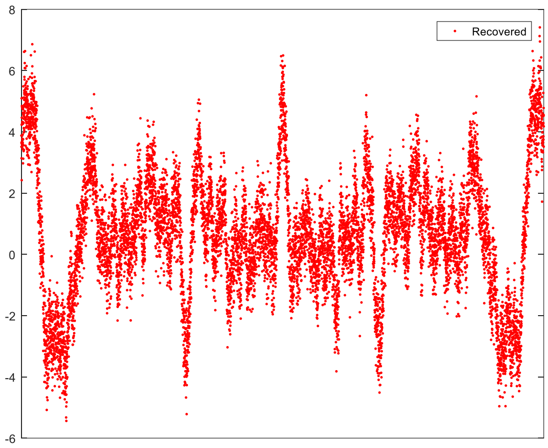

Fig.13 Reconstructed Beam Pattern Using Compressed Perception Norm Method

4. Experimental Results and Analysis

Table 1 shows the amplitude values of the original received signals of the antenna array.

Tab.1 Amplitude Values of Original Signal Full Array Elements

signal amplitude（V）									
-0.0311	-0.1925	0.4053	0.5648	0.7426	0.1714	0.4891	0.3297	0.6656	0.6047
0.3475	-0.4174	0.0954	0.3058	0.0802	0.4134	0.4459	0.6422	0.7867	0.4735
0.8189	-0.0103	1.0585	0.5341	1.1915	0.8083	2.6089	0.3167	4.9903	9.8292
9.6832	5.2282	0.0383	2.5978	0.6523	1.3942	1.1906	1.3611	0.4705	1.0674
0.5760	0.6019	0.4613	0.5057	-0.0188	0.0364	0.3089	0.4767	-0.5791	0.5235
0.5121	0.5891	0.0670	0.9769	0.2371	0.7342	0.3798	0.3554	0.0154	-0.0766

The signal amplitude of the sparse antenna array after sparse representation of the array is obtained by projecting the above array on a sparse basis of discrete cosine transform, as shown in Table 2.

Tab.2 Sparse Representation of Arrays

Sparse signal amplitude（V）									
7.4738	-5.6667	-4.3973	3.7026	4.4928	-3.4291	-3.0496	1.8445	3.1862	-2.1076
-3.1862	-2.1076	-3.9561	1.3025	2.9388	-1.0853	-2.8716	1.1721	2.6560	-1.1049
-2.9928	0	2.2084	0	-2.8284	0	1.8451	0	-1.6828	0
2.0666	0	-1.6870	0	0	0	-1.5257	0	1.4382	0
0	0	0	0	0	0	0	0	0	0
0	0	-1.1739	0	1.3518	0	0	0	1.1027	0

The beamforming diagram based on compressed sensing is shown in Figure 14.

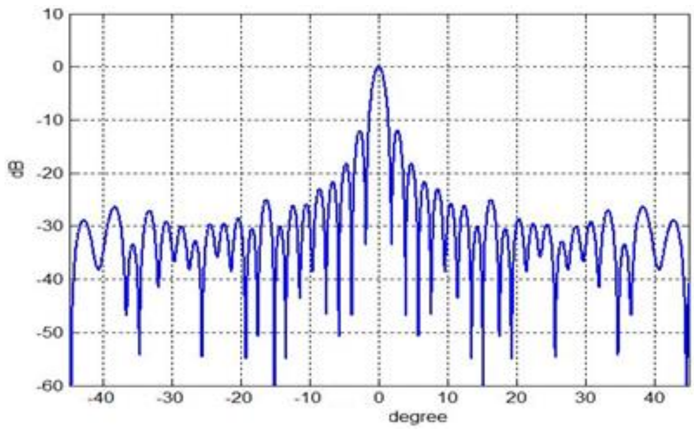

Fig.14 Beamforming based on compressed sensing

Fig.14 shows the results of using compressed sensing theory to reconstruct the smart antenna array signal when the array is full, and then using the recovered signal feature parameters for beamforming. From the figure, it can be seen that the formed beam has accurate main lobe direction in the useful signal direction with an incidence angle of 0 °, zero notch depth in the interference signal direction, and a side lobe gain as low as -30dB, which is consistent with the performance when the array is full.

5. Conclusion

This article studies beamforming technology based on compressed sensing, and proves the effectiveness of beamforming technology based on compressed sensing through relevant experimental analysis. The conclusion is as follows:

(1) The algorithm in this article has good performance in beamforming.

(2) The improved beam formed by compressed sensing optimizes system performance and greatly

reduces sidelobe error.

(3) The compressed sampling and reconstruction process of a composite sine signal as the target signal under random undersampling conditions has been well achieved, which has certain promotion significance and research value.

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